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Vector boson Pair Production at Supercolliders; useful approximate helicity amplitudes†

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Abstract

We study vector boson pair production at LHC and SSC, taking into account the effects generated by the anomalous vector boson and Higgs couplings induced by the operators $\mathcal{O}_W$ and $\mathcal{O}_{UW}$, which are the only dim=6 operators preserving $SU(2)_c$. These operators lead to enhanced production of transverse vector bosons, as opposed to the enhanced production of longitudinal gauge bosons, induced in case $M_H \gtrsim 1\ TeV$, by dim=4 terms already existing in the Standard Model lagrangian. For vector boson pair masses larger than 1 TeV, we establish very simple approximate expressions for the standard as well as the non-standard helicity amplitudes for $q\bar{q}$ annihilation and vector boson fusion, which accurately describe the physics. These expressions should simplify the experimental search for such interactions. We finally discuss the observability and the disentangling of these interactions.

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1 Introduction

Studies of the gauge sector couplings constitute an important part of the program of tests of the Standard Model and of the search for New Physics (NP) beyond it. Such a study is particularly important if the new particles connected to NP turn to be so heavy, that they cannot actually be produced. Recently it has been recognized that indirect tests [1,2] using high precision LEP 1 results at Z peak, are far from having the required accuracy for a thorough study of NP, and in any case they can only be meaningful for specific forms of the new physics models. Thus, direct measurements using vector boson pair production have been shown to be essential for a comprehensive study of the gauge boson (and Higgs) couplings induced by NP [3-5]. Such direct tests will be first achieved at LEP 2, where the study of $e^+e^- \to W^+W^-$ could provide rough upper bounds of the order of 10 percent, on the strength of the various three gauge boson couplings [3]. Later on, a 500 GeV collider could lower these bounds to the order of 1 percent [4]. In between, vector boson pair production in pp collisions at LHC or SSC, should also be able to supply information on the anomalous gauge boson and Higgs couplings, at the same level of accuracy [6].

Departures from the standard Yang-Mills couplings (corrected by Standard Model (SM) loop effects) could arise from various sources like e.g. high order effects involving new heavy particles, mixing with higher vector bosons, genuine departures from SM structure, etc. Loop corrections are generally expected [7-9] to lead to very small contributions to these departures, marginally reaching the aforementioned 1 percent level.

However, if the bosonic sector turns out to be strongly interacting, then various processes may be enhanced [10]. This is a very interesting case, provided the bosonic sector is weakly coupled to fermions, so that it avoids the constraints from LEP 1 measurements. In a phenomenological analysis, a "natural" way of selecting the NP operators that would not affect fermions may come from a symmetry principle. In a previous paper [6], we have emphasized that the global $SU(2)_c$ custodial symmetry provides a very interesting such principle, which can be used to guide our search for NP beyond SM. Such a symmetry is of course not new. It has been invoked in the past in order to justify the famous result $\rho \simeq 1$. It was also used later, together with the implicit assumption that at present energies NP only affects the purely scalar sector [11]. On the basis of these assumptions it was concluded that, in case $M_H \gtrsim 1TeV$, NP induces a new strong interaction among the <u>longitudinal</u> W and Z gauge bosons only [11].

In Ref. [6] we followed another root in exploring the $SU(2)_c$ consequences, and allowed the $SU(2)_c$ invariant field $\overrightarrow{W}_\mu$, to participate in the NP induced operators. We then concluded that, provided NP is $SU(2)_c$ and CP symmetric and satisfies $SU(2) \times U(1)$ gauge invariance, it can also induce the dim=6 operators dubbed $\mathcal{O}_W$ and $\mathcal{O}_{UW}$ dimension. As we will see below, $\mathcal{O}_W$ involves couplings among the gauge boson fields $W_{\mu\nu}$ only, whereas $\mathcal{O}_{UW}$ describes an interaction containing the Higgs field also. In both cases, the appearence of $W_{\mu\nu}$ guarantees that NP will predominantly enhance the production of <u>transverse</u> W states at high energies [6], as opposed to the enhancement of the longitudinal production considered before [11]. More explicitely, $\mathcal{O}_W$ and $\mathcal{O}_{UW}$ will always enhance transverse-transverse (TT) pair production in e^+e^- and $q\bar{q}$ annihilation as well as in

boson-boson fusion. Depending on the magnitude of the relevant couplings, these new interactions may even induce a new strong force among the transverse gauge bosons. We have found that the experimental search for such interactions is much easier than the study of the strong production of longitudinal gauge bosons, arising in case the physical Higgs particle is very heavy [6]. This is because in the $\mathcal{O}_W$ and $\mathcal{O}_{UW}$ cases, it is sufficient to study the total rate of vector boson pair production, with no need for large background subtraction.

According to Ref. [6] and the expected luminosities at LHC or SSC, the SM rates of vector boson pair production should be observable for invariant masses of the vector pair in the range M=1 to 2 TeV. In this range, departures from SM due to an $\mathcal{O}_W$ anomalous coupling of the order of a percent, should be easily observable at SSC/LHC [6].

In this paper we first show that for vector pairs in the TeV range, there exist very simple approximate expressions for all $q\bar{q}$ and vector fusion helicity amplitudes, corresponding to both the standard and the non-standard interactions. These amplitudes are accurate at the percent level and should be very useful for the experimental analysis of the sensitivity to anomalous couplings. We then study separately the effects of the operator $\mathcal{O}_W$ with a coupling λ_W, and $\mathcal{O}_{UW}$ with coupling d, on $q\bar{q}$ annihilation and boson-boson fusion.

For studying the observability at SSC/LHC, we sum up all processes for a given final state, and we ask for a 50 percent change in the rate of the mass distribution $d\sigma/dM$ in the 1 to 2 TeV range in order to conclude that we indeed have a signal of New Physics (NP). A first study of the $\mathcal{O}_W$ only has already been done in [6]. Here we add the Higgs exchanges contributions to boson-boson fusion due to $\mathcal{O}_{UW}$, and we study the possibility to disentangle the effects of the two operators. We find that disentangling is in principle possible, since different classes of final states respond differently to λ_W and d. We find for example that the ratios WZ/ZZ and $W\gamma/ZZ$ respond in opposite ways as we increase $|\lambda_W|$ and $|d|$. As a result, the discovery limits appear to be of the order of 0.01 for $|\lambda_W|$, and 0.1 for $|d|$.

The plan of the paper is the following. In Section 2 we recall the gauge invariant forms of the $SU(2)_c$ preserving operators to be added to the Standard Model lagrangian and give the corresponding 3-boson and 4-boson vertices. The implied simple approximate expressions for the $q\bar{q}$ annihilation and boson-boson fusion amplitudes to two final gauge bosons are given in the Appendices A and B. The distribution $d\sigma/dM$ is expressed using the parton model, in terms of the quark and vector boson distribution functions of the proton and the subprocess cross sections. The results are presented in Section 3 for LHC or SSC energies, and invariant masses of the final vector pair in the range of 1 to 2 TeV. The observability of λ_W and d effects is also discussed. Finally, Section 4 summarizes the results.

2 $SU(2)_c$ preserving couplings and subprocess amplitudes

It has been shown in [6] that, if we impose CP and $SU(2)_c$ symmetries as well as $SU(2) \times U(1)$ gauge invariance and we restrict to operators of dimension up to six, then the effective lagrangian describing gauge boson production at the various colliders is completely described by

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{NP} \quad , \tag{1}$$

$$\begin{aligned} \mathcal{L}_{SM} &= -\frac{1}{2}\langle W_{\mu\nu} W^{\mu\nu}\rangle - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger \rangle \\ & -\frac{v^2 M_H^2}{8}\left(\frac{1}{2}\langle U U^\dagger\rangle - 1\right)^2 + \text{ fermionic terms} \quad , \end{aligned} \tag{2}$$

$$\mathcal{L}_{NP} = \lambda_W \frac{g_2}{M_W^2} \mathcal{O}_W + d\mathcal{O}_{UW} \quad , \tag{3}$$

where

$$\mathcal{O}_W = \frac{1}{3!}\left(\overrightarrow{W}_\mu{}^\nu \times \overrightarrow{W}_\nu{}^\lambda\right) \cdot \overrightarrow{W}_\lambda{}^\mu = -\frac{2i}{3}\langle W^{\nu\lambda} W_{\lambda\mu} W^\mu{}_\nu \rangle \quad , \tag{4}$$

$$\mathcal{O}_{UW} = \langle (UU^\dagger - 1)\ W^{\mu\nu}\ W_{\mu\nu}\rangle \quad , \tag{5}$$

and the standard Higgs doublet is given by

$$U = (\ \widetilde{\Phi}\ \ ,\ \Phi\ )\ \frac{\sqrt{2}}{v} \quad , \tag{6}$$

$$\Phi = \begin{pmatrix} \phi^\dagger \\ \frac{1}{\sqrt{2}}(v + H + i\phi^0) \end{pmatrix} \quad . \tag{7}$$

Here the definitions $\langle A \rangle \equiv TrA$ and $\widetilde{\Phi} = i\tau_2 \Phi^*$ are used. Contrary to the heavy higgs scenario studied in [11-13], in the present work we address the case that the physical Higgs particle is rather light; i.e. $M_H \lesssim 1TeV$. Thus, as explained in the Introduction, we concentrate on the possibility that the new $SU(2)_c$ invariant forces are realized via the transverse gauge boson components. This can be seen explicitely from the new three- and four boson vertices generated in the unitary gauge by $\mathcal{L}_{NP}$ for VW^+W^-

$$i\lambda_W \frac{g_2}{M_W^2}\ W^3_{\nu\mu}\ W^{-\mu\lambda}\ W^+{}_\lambda{}^\nu \quad , \tag{8}$$

for $VV'W^+W^-$

$$\lambda_W \frac{g_2^2}{M_W^2} \{ (W_\mu^- W_\nu^+ - W_\mu^+ W_\nu^-) \, W^{-\nu\lambda} \, W^+{}_\lambda{}^\mu$$
$$- W^3_{\mu\nu} W^3_\lambda \, (W^{+\nu} W^{-\lambda\mu} + W^{-\nu} W^{+\lambda\mu})$$
$$+ W^3_{\mu\nu} W^{3\nu} \, (W^+{}_\lambda W^{-\lambda\mu} + W^-{}_\lambda \, W^{+\lambda\mu}) \} \quad , \tag{9}$$

and for HW^+W^-

$$d \frac{g_2}{M_W} \, H \quad \{ \, W^+{}_{\mu\nu} \, W^{-\mu\nu}$$
$$+ \frac{1}{2} (c_W^2 \; Z_{\mu\nu} \; Z^{\mu\nu} + s_W^2 \; F_{\mu\nu} \; F^{\mu\nu} + 2 s_W c_W \; F_{\mu\nu} \; Z^{\mu\nu}) \} \; , \tag{10}$$

where g_2 is the usual $SU(2)$ gauge coupling and $W^3_\mu = c_W Z_\mu + s_W A_\mu$.

In pp collisions two types of subprocesses can produce vector boson pairs, namely $q\bar{q}$ annihilation and boson-boson fusion [6]. The $q\bar{q}$ annihilation to a pair of gauge bosons proceeds through standard model quark exchange, and through W,Z,γ formation diagrams. Only the later ones are sensitive to λ_W contributions arising from three gauge boson couplings. Although complete expressions for the sub-cross sections $d\sigma/dcos\theta$ were given in [6], it is instructive to look at the helicity amplitudes of this subprocess.

In the high energy approximation (which has been checked to be valid at the percent level for vector pair masses M in the TeV range), the $q\bar{q} \to VV'$ amplitudes aquire the very simple form given in Appendix A. From this we remark that at high energies the SM contribution to the amplitudes becomes energy-independent, while the $\mathcal{O}_W$ contribution is proportional to $\lambda_W s/M_W^2$, in agreement with naive expectations based on dimensional arguments and the equivalence theorem [12]. It can also be seen that for the $q\bar{q}$ annihilation at high energies, there is never any intereference between the λ_W and SM contributions, since $\mathcal{O}_W$ contributes only to amplitudes involving final gauge bosons with equal transverse helicities, for which the SM contributions vanish. This implies that the cross section for the $q\bar{q} \to VV'$ subprocess is always quadratic in λ_W. Consequently, the results for the $q\bar{q}$ annihilation to any gauge boson pair presented in Figs. 4 to 11 of [6] for $\lambda_W = 0.01$, are also valid for $\lambda_W = -0.01$. Thus, these figures indeed provide a feeling of the general behaviour of a non-vanishing $\mathcal{O}_W$ contribution for any sign of λ_W.

We now turn to boson-boson fusion processes. These proceed through vector boson exchanges and contact interactions, and in many cases also through Higgs boson exchanges. They contain therefore λ_W as well as d contributions. In [6], we numerically discussed only the λ_W effects, using the exact tree level formulae of the SM and $\mathcal{O}_W$ contributions. Explicit expressions for the corresponding vector fusion amplitudes were not given though, because they were extremely lengthy. However we have found that for vector pair masses

M in the TeV range, very simple approximate expressions for all the vector fusion helicity amplitudes exist, which are accurate at the percent level. They are given in Appendix B, and are very useful for the phenomenological analysis of both standard and non standard contributions.

It is instructive to contemplate on the way λ_W and d contribute to these amplitudes. We first turn to the SM and $\mathcal{O}_W$ contributions and remark that they appear in the channels $\gamma W \to \gamma W,\ \gamma W \to ZW,\ ZW \to \gamma W,\ \gamma\gamma \to WW,\ \gamma Z \to WW,\ WW \to \gamma\gamma,$ $WW \to \gamma Z,\ ZZ \to WW,\ WW \to ZZ,\ ZW \to ZW$ and $WW \to WW$ channels. There exist only three different types of terms contributing to these channels; namely purely SM terms that are energy-independent at high energies, and terms proportional either to $\lambda_W s/M_W^2$ or $(\lambda_W s/M_W^2)^2$, with their coefficients depending only on the CM scattering angle θ for the sub-process. As in the annihilation case, such a result is intuitively expected on the basis of naive dimensional arguments and the equivalence theorem [12]. According to the results in Appendix B, at most only two of these types of terms can contribute in the same helicity amplitude, creating rather small interferences effects and a weak sensitivity of the sub-process cross section on the sign of λ_W. This sensitivity largely disappears though, when we consider ratios of production cross sections for the various two gauge boson channels; see below.

We next turn to the $\mathcal{O}_{UW}$ contribution determined by the coupling d. Such a contribution exists for all vector fusion channels. In particular it exists also for the pure neutral channels $\gamma\gamma \to \gamma\gamma,\ \ \gamma Z \to \gamma\gamma,\ \ ZZ \to \gamma\gamma,\ \ \gamma\gamma \to \gamma Z,\ \ \gamma Z \to \gamma Z,\ \ ZZ \to \gamma Z,\ \ \gamma\gamma \to ZZ,\ \gamma Z \to ZZ$, which receive neither SM nor $\mathcal{O}_W$ contributions at tree level. In the asymptotic formulae given in Appendix B, no assumption on the Higgs mass was made. Nevertheless, if the Higgs mass is comparable to M_W, these amplitudes acquire an even simpler asymtotic form expressed in terms of two contributions proportional to ds/M_W^2 and $d^2 s/M_W^2$ respectively, with their coefficients depending on the scattering angle in the c.m. of the sub-process.

It also turns out that the $\mathcal{O}_{UW}$ contribution is invisibly small unless $|d| \gtrsim 0.1$, for which the quadratic d^2 term dominates the amplitudes, and any sensitivity of the production cross sections on the sign of d is lost. Also worth noting from the amplitudes in Appendix B is the fact, that this d^2 term contributes mainly to the production of transverse gauge bosons. Thus, provided $M_H \sim M_W$, the contribution of any of the two $SU(2)_c$ induced operatos $\mathcal{O}_W$ and $\mathcal{O}_{UW}$, is either negligible or affects mainly the transverse gauge bosons.

3 Observability at LHC or SSC

We have computed $d\sigma/dM$ for pp collisions as explained in [6]. Subprocess cross sections are convoluted with quark or vector boson distribution functions inside the proton. The same cuts on the rapidity Y and the photon transverse momentum are applied; i.e. $|Y| \leq 2$, $|p_{\gamma T}| \leq 0.05 TeV$. For a given final vector boson pair we sum all processes with different initial states. In particular we note the importance of including initial states containing photons as now transverse states are dominant. As mentioned in Section 1 there is now

essentially no background problem as the signal we are looking for affects all states and predominantly the leading TT ones.

Consequently, the experimental search for the $\mathcal{O}_W$ and $\mathcal{O}_{UW}$ effects discussed here, is much simpler than the search for the strong interactions affecting the longitudinal gauge bosons discussed in [11,13,14]. In this later case the interesting signal is completely contained in the LL final states, whose study requires the subtraction of a large SM background due to the transverse gauge boson productions in $q\bar{q}$ annihilation and vector fusion sub-processes.

One could also worry about a possible background from gluon-gluon fusion processes that can contribute, through a quark loop, to the SM result for W^+W^-, ZZ, $Z\gamma$ and $\gamma\gamma$ production. However, the results of [15] show that for $M_H \sim M_W$, $m_t = 120GeV$, and $M_{ZZ} \gtrsim 1TeV$ we have

$$\frac{d\sigma(gg \to ZZ)/dM_{ZZ}}{d\sigma(q\bar{q} \to ZZ)/dM_{ZZ}} \lesssim 0.25 \text{ for SSC} \quad . \tag{11}$$

For LHC the r.h.s. of the inequality (11) becomes ~ 0.1, and in both (SSC and LHC) cases it slowly decreases as M_{ZZ} increases. These results are understandable on the basis of the softness of the gluon distribution functions, and allow us to expect that the gluon fusion background should not be very important for $M \gtrsim 1TeV$. In any case, we have neglected this SM background in the present work.

There are certainly uncertainties introduced by the quark and vector boson distribution functions inside the proton. For this reason we define the discovery limit for an $\mathcal{O}_W$ or $\mathcal{O}_{UW}$ interaction, by asking for a 50 percent change in the rate as compared to the SM prediction. This is what gives the order of magnitude limits of 0.01 on $|\lambda_W|$ and 0.1 for $|d|$. We also stress the fact that ratios of production rates to different final states, should be less sensitive to the uncertainties in distribution functions.

We also emphasize that in the present case, where all helicities of the final (and initial) gauge bosons are summed over, the $q\bar{q}$ annihilation and vector fusion mechanisms are both important, for producing gauge boson pairs in the range $1TeV \lesssim M \lesssim 2TeV$. This result was first noticed in [6] and it contrasts sharply with the situation in Refs.[13,14], where the interest is focused to the production of longitudinal gauge bosons at lower M values and vector fusion is ignored.

We now turn to the comparison of $\mathcal{O}_W$ and $\mathcal{O}_{UW}$ effects. We classify the various contributing channels into three classes:

a) Channels in which the enhancements for the limiting values $|\lambda_W| = 0.01$ and $|d| = 0.1$ are similar, like e.g. $WZ \to WZ, W\gamma \to W\gamma$. Fig.1 illustrates the case of $W^+Z \to W^+Z$.

b) Channels in which the $|d| = 0.1$ effects are larger than $|\lambda_W| = 0.01$ ones. This happens when the standard contribution is depressed or absent, like e.g. in the purely neutral channels as $ZZ \to ZZ$, $\gamma\gamma \to \gamma\gamma$, $\gamma Z \to \gamma Z$, where λ_W does not contribute. Fig.2 illustrates the ZZ case.

c) Finally we consider channels in which d effects are weaker than λ_W ones. This is

the case of the WW channels, like $\gamma\gamma \to WW$, $WW \to WW$, because of the presence of large photon contributions and strong charged W couplings.

Fig.3 and 4 show the results of summing all initial states for given final states. As expected from the amplitudes given in Appendix A and B, changing the sign of λ_W and d leads to similar effects. So roughly no ambiguity should arise from these signs, in the 1 to 2 TeV range of M.

With the expected LHC or SSC luminosities and the need to observe at least 10 events in order to establish NP, the figures confirm the discovery limits that we have announced. The disentangling of λ_W and d can be done by comparing departures to Standard predictions in the three above classes of channels. A spectacular way of illustrating these properties consists in plotting specific ratios, like those presented in Figs. 5 and 6. As we see there, irrespective of signs a remarkable separation of the λ_W and d effects is provided by the ratios WZ/ZZ and $W\gamma/ZZ$, which can further be tested by also studying ratios like $WZ/\gamma\gamma$, $WZ/W\gamma$.

4 Final discussion

We have examined the observable consequences of the existence of New Physics in the gauge boson sector that preserves $SU(2)_c$ global symmetry in addition to the $SU(2)\times U(1)$ spontanously broken gauge invariance. This is a natural way to allow for non standard self-boson couplings that avoid strong constraints from high precision tests at LEP 1. Two effective interactions $\lambda_W \mathcal{O}_W$ and $d\mathcal{O}_{UW}$ modify vector boson self-couplings and Higgs-vector boson couplings respectively. They lead to TT dominance at high energies, a complementary case to the one generated by the $SU(2)_c$ conserving scalar sector leading to LL dominance. In the TT case there is no background problem contrarily to the LL case. We have then discussed the search of such interactions at LHC or SSC.

The first main result of this paper is the establishment of very simple and useful high energy approximate expressions for the helicity amplitudes of all subprocesses with $q\bar{q}$ annihilation and boson-boson fusion. These expressions allow us to understand the features of λ_W and d effects and to compute them very easily, with an accuracy of 1 percent for $1TeV \lesssim M \lesssim 2TeV$. They should be very useful for phenomenological analyses in pp and e^+e^- collisions.

Our second main result concerns the disentangling of λ_W and d effects. Summing over $q\bar{q}$ and boson-boson initial states we have found three classes of final states characterized by their M dependence for various λ_W and d. The most spectacular such behaviour is observed in the ratios WZ/ZZ and $W\gamma/ZZ$ shown in Fig.6. As we see there, λ_W and d give opposite departures to the Standard Model predictions, irrespective of the signs of the couplings. These ratios can therefore be used in order to disentangle λ_W and d contributions. Other ratios can be used in order to further test whether this disentangling is correct.

The discovery limit of 0.01 for λ_W and 0.1 for d have different implications. In the pure gauge sector, such λ_W measurement at SSC or LHC should improve by an order

of magnitude the anticipated LEP2 result, and by two orders of magnitude the result from LEP1. Thus the SSC/LHC accuracy is comparable to the one expected from NLC. However the new feature here is the possibility to study many different channels, and to have access to the other $SU(2)_c$ conserving operator $\mathcal{O}_{UW}$, which affects Higgs couplings not directly accessible in e^+e^-. Concerning the $\mathcal{O}_{UW}$ effects, we should also remark that although the accuracy on d is weaker than the one on λ_W, it will constitute an important insight to the Higgs sector. Finally, the complementarity to the $SU(2)_c$ LL interactions, should allow to get a view of the structure of the interactions associated to the mechanism of mass generation, that is still a most puzzling domain in particle physics.

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Appendix A : Helicity amplitudes for $q\bar{q} \to VV'$ processes at High Energy

Neglecting quark masses, the quark annihilation amplitudes vanish, unless q and $\bar{q}$ have opposite helicities denoteted by λ and $-\lambda$ respectively. The helicities of the vector states V and V' are denoted by τ and τ' respectively. The helicity amplitudes for the various annihilation processes $q(\lambda)\bar{q}(-\lambda) \to V(\tau)V'(\tau')$ are described by $F^L_{\tau\tau'}$ for $\lambda = 1/2$, and by $F^R_{\tau\tau'}$ for $\lambda = -1/2$. The Jacob-Wick phase conventions [16] are used, and the antifermion wave function is such that $u(k,\pm) = C\bar{v}^\tau(k,\pm)$; i.e. charge conjugation implies e^-_L $(e^-_R) \to e^+_L$ (e^+_R) exactely. Imposing CP invariance, there are at most 6 independent $F^L_{\tau\tau'}$ helicity amplitudes, and another 6 $F^R_{\tau\tau'}$ ones. The normalization is such that the differential cross section is given by

$$\frac{d\sigma}{dcos\theta} = C\sum_{\lambda\tau\tau'}|F_{\lambda\tau\tau'}|^2 \tag{A.1}$$

where the coefficient

$$C = \frac{1}{128N_c\pi s}\frac{2p_V}{\sqrt{s}} \tag{A.2}$$

takes care of the average over the initial $q\bar{q}$ spin states and colour factor N_c. Here and below the usual Mandelstam variables for the sub-processes are denoted by s, t, u, while p_V is the cm momentum of the final bosons and θ is the angle between W^- and the fermion. Q is the fermion charge in units of e, and $\tau_3 = -1$ if the initial quark is d, and $\tau_3 = +1$ if it is u. The amplitudes for the various channels for $s \gtrsim 1TeV^2$ are:

$\boxed{e^+e^-,\ d\bar{d},\ u\bar{u} \to W^+W^-}$

$$\begin{aligned}
F^L_{++} &= F^L_{--} = \tau_3\frac{e^2}{4s^2_W}\left(\frac{\lambda_W s}{M^2_W}\right)\sin\theta \\
F^L_{+-} &= -\frac{e^2}{2s^2_W}\sin\theta\frac{(1-\tau_3\cos\theta)}{1+\cos\theta} \quad ; \quad F^L_{-+} = \frac{e^2}{2s^2_W}\sin\theta\frac{(1-\tau_3\cos\theta)}{1-\cos\theta} \\
F^L_{LL} &= \tau_3\frac{e^2}{2c^2_W}\sin\theta\,(|Q|-1+\frac{1}{2s^2_W}) \\
F^R_{LL} &= Q\frac{e^2}{2c^2_W}\sin\theta
\end{aligned} \tag{A.3}$$

$\boxed{d\bar{u} \to W^- Z}$

$$
\begin{aligned}
F^L_{++} &= F^L_{--} = -\frac{e^2}{2\sqrt{2}}\frac{c_W}{s_W^2}\left(\frac{\lambda_W s}{M_W^2}\right)\sin\theta \\
F^L_{+-} &= -\frac{e^2}{\sqrt{2}c_W s_W^2}\;\frac{\sin\theta}{1+\cos\theta}\;(\,c_W^2\cos\theta - \frac{s_W^2}{3}\,) \\
F^L_{-+} &= \frac{e^2}{\sqrt{2}c_W s_W^2}\;\frac{\sin\theta}{1-\cos\theta}\;(\,c_W^2\cos\theta - \frac{s_W^2}{3}\,) \\
F^L_{LL} &= -\frac{e^2}{2\sqrt{2}\,s_W^2}\;\sin\theta
\end{aligned}
\tag{A.4}
$$

$\boxed{d\bar{u} \to W^- \gamma}$

$$
\begin{aligned}
F^L_{++} &= F^L_{--} = -\frac{e^2}{2\sqrt{2}\,s_W}\sin\theta\left(\frac{\lambda_W s}{M_W^2}\right) \\
F^L_{+-} &= -\frac{e^2}{\sqrt{2}\,s_W}\;\frac{\sin\theta}{1+\cos\theta}\;(\,\cos\theta + \frac{1}{3}\,) \\
F^L_{-+} &= \frac{e^2}{\sqrt{2}\,s_W}\;\frac{\sin\theta}{1-\cos\theta}\;(\,\cos\theta + \frac{1}{3}\,)
\end{aligned}
\tag{A.5}
$$

$\boxed{e^+e^-,\ d\bar{d},\ u\bar{u} \to \gamma\gamma}$

$$
\begin{aligned}
F^L_{+-} &= F^R_{-+} = 2e^2Q^2\,\frac{(\cos\theta - 1)}{\sin\theta} \\
F^L_{-+} &= F^R_{+-} = 2e^2Q^2\,\frac{(\cos\theta + 1)}{\sin\theta}
\end{aligned}
\tag{A.6}
$$

$\boxed{e^+e^-,\ d\bar{d},\ u\bar{u} \to Z\gamma}$

$$
F^L_{+-} = \frac{e^2 Q}{s_W c_W}\,(\tau_3 - 2Qs_W^2)\,\frac{(\cos\theta - 1)}{\sin\theta}
$$

$$
\begin{aligned}
F^L_{-+} &= \frac{e^2 Q}{s_W c_W}(\tau_3 - 2Q s_W^2)\,\frac{(\cos\theta+1)}{\sin\theta} \\
F^R_{-+} &= \frac{2e^2 Q^2 s_W}{c_W}\,\frac{(\cos\theta-1)}{\sin\theta} \\
F^R_{+-} &= \frac{2e^2 Q^2 s_W}{c_W}\,\frac{(\cos\theta+1)}{\sin\theta}
\end{aligned}
\tag{A.7}
$$

$\boxed{e^+e^-,\ d\bar{d},\ u\bar{u} \to ZZ}$

$$
\begin{aligned}
F^L_{+-} &= \frac{e^2}{2 s_W^2 c_W^2}(\tau_3 - 2Q s_W^2)^2\,\frac{(\cos\theta-1)}{\sin\theta} \\
F^L_{-+} &= \frac{e^2}{2 s_W^2 c_W^2}(\tau_3 - 2Q s_W^2)^2\,\frac{(\cos\theta+1)}{\sin\theta} \\
F^R_{-+} &= -\frac{2e^2 Q^2 s_W^2}{c_W^2}\,\frac{(\cos\theta-1)}{\sin\theta} \\
F^R_{+-} &= -\frac{2e^2 Q^2 s_W^2}{c_W^2}\,\frac{(\cos\theta+1)}{\sin\theta}
\end{aligned}
\tag{A.8}
$$

According to (A3)-(A8), $\mathcal{O}_W$ contributions at high energies exist only for the amplitudes F_{++} and F_{--}, where both final gauge bosons have equal and transverse heleicities. These $\mathcal{O}_W$ contributions behave like $\lambda_W s/M_W^2$ to leading order. Since these amplitudes receive no standard contribution, there exists no appreciable interference between standard and λ_W terms at high energies. This explains the lack of sensitivity to the sign of λ_W in these processes.

Appendix B : Helicity amplitudes for boson-boson fusion processes at High Energy

In general there are 81 helicity amplitudes $F_{\lambda\lambda'\mu\mu'}$ for each vector boson-vector boson fusion process $V_1(\lambda)V_2(\lambda') \to V_3(\mu)V_4(\mu')$. Taking into account parity conservation, (which is valid at tree level for the self-boson interactions contained in SM and $\mathcal{O}_W$, $\mathcal{O}_{UW}$), implies the relation

$$F_{\lambda\lambda'\mu\mu'}(\theta) \;=\; F_{-\lambda-\lambda'-\mu-\mu'}(\theta) \quad , \tag{B.1}$$

which reduces the number of independent amplitudes to 41. In specific channels this number is further reduced due e.g. to the absence of helicity zero states for photons, the symmetrization for identical particles, charge conjugation relations, etc. Here and below θ is the angle between V_1 and V_3. The normalization of these amplitudes is defined by noting that the differential cross section in c.m. is given by

$$\frac{d\sigma(\lambda\lambda'\mu\mu')}{dcos\theta} = C|F_{\lambda\lambda'\mu\mu'}|^2 \quad , \tag{B.2}$$

where the coefficient

$$C = \frac{1}{32\pi s}\frac{p_{34}}{p_{12}} \tag{B.3}$$

includes <u>no spin average</u>. This later choice is motivated by the fact that, inside the proton, different vector boson distribution functions occur for different initial helicity states. Finally p_{12}, p_{34} in (B.3) denote the cm momenta of the initial and final boson pairs respectively.

1 Standard and $\mathcal{O}_W$ contributions

In this part we give the SM and $\mathcal{O}_W$ contributions to the amplitudes arising from exchanges of gauge bosons, as well from exchanges of a Higgs boson with $M_H = M_W$. The corrections to be added to these expressions, due to the fact that in general $M_H \neq M_W$, are given in the second part of this Appendix together with the $\mathcal{O}_{UW}$ contributions. The motivation for this choice is that it leads to SM contibutions which are always finite, even in the presence of loops. We thus have for $s \gtrsim 1TeV^2$:

$\boxed{\gamma W \to \gamma W}$

$$F_{++++} \;=\; F_{----} = -\, e^2 \left\{ \frac{4}{1+\cos\theta} + \left(\frac{\lambda_W s}{M_W^2}\right)^2 \frac{\cos\theta}{4} \right\}$$

$$F_{+++-} \;=\; F_{++-+} = F_{+-++} = F_{-+++} =$$

$$
\begin{aligned}
F_{---+} &= F_{--+-} = F_{-+--} = F_{+---} = e^2\,\frac{(1-\cos\theta)}{2}\,\left(\frac{\lambda_W s}{M_W^2}\right)\\
F_{+--+} &= F_{-++-} = -\,e^2\,\frac{(1-\cos\theta)^2}{1+\cos\theta}\\
F_{+-+-} &= F_{-+-+} = -\,e^2\,(1+\cos\theta)\left\{1+\frac{3-\cos\theta}{16}\,\left(\frac{\lambda_W s}{M_W^2}\right)^2\right\}\\
F_{++--} &= F_{--++} = e^2\,\left(\frac{\lambda_W s}{M_W^2}\right)\left\{1-\cos\theta-\frac{(3+6\cos\theta-\cos^2\theta)}{16}\,\left(\frac{\lambda_W s}{M_W^2}\right)\right\}\\
F_{+L+L} &= F_{-L-L} = -2e^2 \qquad \text{(B.4)}
\end{aligned}
$$

$\boxed{\gamma W \to ZW}$

The purely transverse amplitudes are identical with those in $\gamma W \to \gamma W$, provided we replace $e^2 \to e^2 c_W/s_W$. The amplitudes involving longitudinal bosons are

$$
\begin{aligned}
F_{++LL} &= F_{--LL} = \frac{e^2}{4s_W}\cos\theta\,\left(\frac{\lambda_W s}{M_W^2}\right)\\
F_{+-LL} &= F_{-+LL} = -\,\frac{e^2}{2s_W}\,(1-\cos\theta)\\
F_{+LL-} &= F_{-LL+} = \frac{e^2}{8s_W}\,(\cos\theta-3)\,\left(\frac{\lambda_W s}{M_W^2}\right)\\
F_{+LL+} &= F_{-LL-} = -\,\frac{e^2}{s_W}\,\frac{(\cos\theta-1)}{\cos\theta+1} \qquad \text{(B.5)}
\end{aligned}
$$

$\boxed{\gamma\gamma \to WW}$

$$
\begin{aligned}
F_{++++} &= F_{----} = \frac{8e^2}{\sin^2\theta}\\
F_{+++-} &= F_{++-+} = F_{+-++} = F_{-+++} =\\
F_{---+} &= F_{--+-} = F_{-+--} = F_{+---} = -e^2\,\left(\frac{\lambda_W s}{M_W^2}\right)\\
F_{+--+} &= F_{-++-} = e^2(1-\cos\theta)\left\{\frac{2}{1+\cos\theta}+\frac{3+\cos\theta}{16}\left(\frac{\lambda_W s}{M_W^2}\right)^2\right\}\\
F_{++--} &= F_{--++} = e^2\left(\frac{\lambda_W s}{M_W^2}\right)\left\{-2\;+\;\frac{3-\cos^2\theta}{8}\left(\frac{\lambda_W s}{M_W^2}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
F_{+-+-} &= F_{-+-+} = -e^2\,(1+\cos\theta)\left\{\frac{2}{\cos\theta-1}+\frac{(\cos\theta-3)}{16}\,\left(\frac{\lambda_W s}{M_W^2}\right)^2\right\} \\
F_{+-LL} &= F_{-+LL} = 2e^2
\end{aligned}
\tag{B.6}
$$

$\boxed{\gamma Z \to WW}$

The purely transverse amplitudes are identical with those in $\gamma\gamma \to WW$ provided we replace $e^2 \to e^2 c_W/s_W$. The amplitudes involving longitudinal bosons are

$$
\begin{aligned}
F_{+LL+} &= F_{-LL-} = -\,\frac{e^2}{s_W}\,\frac{2}{1+\cos\theta} \\
F_{+L-L} &= F_{-L+L} = -\,\frac{e^2}{8s_W}\,(3+\cos\theta)\,\left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{+L+L} &= F_{-L-L} = \frac{e^2}{s_W}\,\frac{2}{1-\cos\theta} \\
F_{+LL-} &= F_{-LL+} = \frac{e^2}{8s_W}\,(3-\cos\theta)\,\left(\frac{\lambda_W s}{M_W^2}\right)
\end{aligned}
\tag{B.7}
$$

$\boxed{ZZ \to WW}$

The purely transverse amplitudes are identical with those in $\gamma\gamma \to WW$ provided we replace $e^2 \to e^2 c_W^2/s_W^2$. The longitudinal amplitudes are given by

$$
\begin{aligned}
F_{L++L} &= F_{L--L} = F_{+LL+} = F_{-LL-} = -\frac{e^2}{s_W^2}\,\frac{(4c_W^2-1-\cos\theta)}{2c_W(1+\cos\theta)} \\
F_{+L-L} &= F_{-L+L} = F_{L-L+} = F_{L+L-} = -\frac{e^2 c_W}{s_W^2}\,\frac{(3+\cos\theta)}{8}\,\left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{L+L+} &= F_{L-L-} = F_{+L+L} = F_{-L-L} = -\frac{e^2}{s_W^2}\,\frac{(1-\cos\theta-4c_W^2)}{2c_W(1-\cos\theta)} \\
F_{+LL-} &= F_{-LL+} = F_{L-+L} = F_{L+-L} = \frac{e^2 c_W}{s_W^2}\,\frac{(3-\cos\theta)}{8}\,\left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{+-LL} &= F_{-+LL} = \frac{e^2}{2}\left(\frac{c_W}{s_W}-\frac{s_W}{c_W}\right)^2 \\
F_{LL+-} &= F_{LL-+} = \frac{e^2}{2s_W^2} \quad ; \quad F_{LLLL} = \frac{e^2}{s_W^2}\,\frac{(5+3cos^2\theta)}{4\sin^2\theta}
\end{aligned}
\tag{B.8}
$$

$\boxed{ZW \to \gamma W, WW \to \gamma\gamma, WW \to \gamma Z \text{ and } WW \to ZZ}$

These amplitudes are respectively identical with those in $\gamma W \to ZW$, $\gamma\gamma \to WW$, $\gamma Z \to WW$ and $ZZ \to WW$ by interchanging helicities between the initial and final state.

$\boxed{ZW \to ZW}$

The purely transverse amplitudes are identical with those in $\gamma W \to \gamma W$ provided we replace $e^2 \to e^2 c_W^2/s_W^2$. The longitudinal amplitudes are

$$
\begin{aligned}
F_{LL\pm\pm} &= F_{\pm\pm LL} = \frac{e^2 c_W}{4s_W^2}\ \cos\theta\ \left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{LL\pm\mp} &= F_{\pm\mp LL} = \frac{e^2}{2c_W}\ (1 + \frac{c_W^2}{s_W^2}\cos\theta) \\
F_{\pm LL\mp} &= F_{L\pm\mp L} = \frac{e^2 c_W}{8s_W^2}\ (\cos\theta - 3)\ \left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{L\pm\pm L} &= F_{\pm LL\pm} = \frac{e^2}{2c_W s_W^2}\left\{1 + 2c_W^2\ \frac{1-\cos\theta}{1+\cos\theta}\right\} \\
F_{\pm L\pm L} &= -\frac{e^2}{2c_W^2 s_W^2}\ (c_W^2 - s_W^2)^2 \quad ; \quad F_{L\pm L\pm} = -\frac{e^2}{2s_W^2} \\
F_{LLLL} &= -\frac{e^2}{s_W^2}\ \frac{(4 + \cos\theta + \cos^2\theta)}{4(1+\cos\theta)}
\end{aligned}
\tag{B.9}
$$

$\boxed{W^+W^- \to W^+W^-}$

$$
\begin{aligned}
F_{\pm\pm\pm\pm} &= \frac{e^2}{s_W^2}\left\{\frac{4}{1-\cos\theta} - \frac{\cos\theta}{4}\ \left(\frac{\lambda_W s}{M_W^2}\right)^2\right\} \\
F_{\pm\mp\mp\mp} &= F_{\mp\pm\mp\mp} = F_{\mp\mp\pm\mp} = F_{\mp\mp\mp\pm} = -\frac{e^2}{2s_W^2}(1+\cos\theta)\ \left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{\pm\mp\mp\pm} &= \frac{e^2}{s_W^2}(1-\cos\theta)\left\{1 + \frac{3+\cos\theta}{16}\ \left(\frac{\lambda_W s}{M_W^2}\right)^2\right\} \\
F_{\pm\pm\mp\mp} &= \frac{e^2}{s_W^2}\ \left(\frac{\lambda_W s}{M_W^2}\right)\left\{-1 - \cos\theta + \frac{3 - 6\cos\theta - \cos^2\theta}{16}\ \left(\frac{\lambda_W s}{M_W^2}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
F_{\pm\mp\pm\mp} &= \frac{e^2}{s_W^2}\frac{(1+\cos\theta)^2}{1-\cos\theta} \\
F_{LL\pm\mp} &= F_{\pm\mp LL} = \frac{e^2}{2s_W^2}(1+\cos\theta) \\
F_{L\pm L\pm} &= F_{\pm L\pm L} = \frac{e^2}{s_W^2}\frac{(1+\cos\theta)}{(1-\cos\theta)} \\
F_{L\pm L\mp} &= F_{\pm L\mp L} = -\frac{e^2}{8s_W^2}(3+\cos\theta)\ \left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{LL\pm\pm} &= F_{\pm\pm LL} = \frac{e^2}{4s_W^2}\cos\theta\ \left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{LLLL} &= -\frac{e^2}{4s_W^2c_W^2}\left\{\ 2c_W^2 + \frac{3+\cos^2\theta}{\cos\theta-1}\ \right\}
\end{aligned}
\tag{B.10}
$$

$\boxed{W^+W^+ \to W^+W^+}$

$$
\begin{aligned}
F_{\pm\pm\pm\pm} &= -\frac{e^2}{s_W^2}\frac{8}{\sin^2\theta} \\
F_{\mp\pm\pm\pm} &= F_{\pm\mp\pm\pm} = F_{\pm\pm\mp\pm} = F_{\pm\pm\pm\mp} = \frac{e^2}{s_W^2}\ \left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{\pm\mp\mp\pm} &= -\frac{e^2}{s_W^2}(1-\cos\theta)\left\{\frac{2}{1+\cos\theta} + \frac{3+\cos\theta}{16}\ \left(\frac{\lambda_W s}{M_W^2}\right)^2\right\} \\
F_{\pm\pm\mp\mp} &= \frac{e^2}{s_W^2}\ \left(\frac{\lambda_W s}{M_W^2}\right)\left\{\ 2\ +\ \frac{\cos^2\theta-3}{16}\ \left(\frac{\lambda_W s}{M_W^2}\right)\right\} \\
F_{\pm\mp\pm\mp} &= -\frac{e^2}{s_W^2}(1+\cos\theta)\left\{\frac{2}{1-\cos\theta} + \frac{3-\cos\theta}{16}\ \left(\frac{\lambda_W s}{M_W^2}\right)^2\right\} \\
F_{L\pm L\pm} &= F_{\pm L\pm L} = -\frac{2e^2}{s_W^2(1-\cos\theta)} \\
F_{\pm LL\pm} &= F_{L\pm\pm L} = \frac{2e^2}{s_W^2(1+\cos\theta)} \\
F_{\pm LL\mp} &= F_{L\pm\mp L} = \frac{e^2}{s_W^2}\frac{(\cos\theta-3)}{8}\ \left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{L\pm L\mp} &= F_{\pm L\mp L} = \frac{e^2}{s_W^2}\frac{(\cos\theta+3)}{8}\ \left(\frac{\lambda_W s}{M_W^2}\right) \\
F_{LLLL} &= \frac{e^2}{2c_W^2}\left\{\ 1 - \frac{4}{s_W^2\sin^2\theta}\ \right\}
\end{aligned}
\tag{B.11}
$$

$\boxed{ZZ \to ZZ}$

$$
\begin{aligned}
F_{LLLL} &= -\frac{3e^2}{4s_W^2} \\
F_{LL\pm\mp} &= F_{\pm\mp LL} = F_{L\pm\pm L} = F_{\pm LL\pm} = -F_{L\pm L\pm} = -F_{\pm L\pm L} = \frac{e^2}{2s_W^2 c_W^2}
\end{aligned}
\quad \text{(B.12)}
$$

2 $\mathcal{O}_{UW}$ contributions, and corrections to the standard model predictions due to $M_H \neq M_W$

It is convenient to express these contributions to the helicity amplitudes as functions of the intial and final helicities. The vector boson helicities are indicated in parentheses below, where ($z = cos\theta$) the vector fusion process is writen as

$$
V_1(\lambda)\ V_2(\lambda') \to V_3(\mu)\ V_4(\mu') \quad . \quad \text{(B.13)}
$$

The masses of the vector bosons are denoted by m_j for $(j = 1, ..., 4)$, while ϵ_1, ϵ_2 denote the polarization vectors for the intials boson states, and ϵ_3, ϵ_4 the complex conjugate ones for the final states. We then use the following definitions at high energies :

$$
\begin{aligned}
(\epsilon_1\epsilon_2) &= -\frac{s}{2m_1m_2}(1-\lambda^2)(1-\lambda'^2) \\
(\epsilon_1\epsilon_3) &= \frac{s(1-\cos\theta)}{4m_1m_3}(1-\mu^2)(1-\lambda^2) \\
(\epsilon_1\epsilon_4) &= -\frac{s(1+\cos\theta)}{4m_1m_4}(1-\mu'^2)(1-\lambda^2) \\
(\epsilon_2\epsilon_3) &= -\frac{s(1+\cos\theta)}{4m_2m_3}(1-\mu^2)(1-\lambda'^2) \\
(\epsilon_2\epsilon_4) &= \frac{s(1-\cos\theta)}{4m_2m_4}(1-\mu'^2)(1-\lambda'^2) \\
(\epsilon_3\epsilon_4) &= -\frac{s}{2m_3m_4}(1-\mu^2)(1-\mu'^2)
\end{aligned}
\quad \text{(B.14)}
$$

$$
\begin{aligned}
V_{12} &= \frac{s}{4}\lambda^2\lambda'^2(1+\lambda\lambda') \quad ; \quad V_{34} = \frac{s}{4}\mu^2\mu'^2(1+\mu\mu') \\
V_{13} &= \frac{s}{8}(1-\lambda\mu)(1-\cos\theta)\mu^2\lambda^2 \quad ; \quad V_{24} = \frac{s}{8}(1-\lambda'\mu')(1-\cos\theta)\mu'^2\lambda'^2 \\
V_{14} &= \frac{s}{8}(1-\lambda\mu')(1+\cos\theta)\mu'^2\lambda^2 \quad ; \quad V_{23} = \frac{s}{8}(1-\lambda'\mu)(1+\cos\theta)\mu^2\lambda'^2
\end{aligned}
\quad \text{(B.15)}
$$

$$Z_{ij} = (\epsilon_i \epsilon_j)\,\frac{M_W}{c_W^2} - \frac{2d}{M_W} c_W^2\ V_{ij} \tag{B.16}$$

The Higgs propagator is written as

$$D_H(x) \equiv x - \mathcal{M}_H^2 \tag{B.17}$$

where $\mathcal{M}_H \equiv M_H^2$ for $x = t$ or u, and $\mathcal{M}_H^2 \equiv M_H^2 - i\ M_H \Gamma_H$ when $x = s$.

Using these definitions, we describe by F_H the sum of the $\mathcal{O}_{UW}$ contributions, and the corrections implied by the SM higgs exhange interactions in case $M_H \neq M_W$. We find

$$\begin{aligned}
F_H(\gamma W \to \gamma W) \ &= \ \frac{2g_2^2 s_W^2 d}{M_W}\, V_{13} \left[\ (\epsilon_2\epsilon_4) M_W - \frac{2d}{M_W} V_{24}\ \right] \frac{1}{D_H(t)} && \text{(B.18)}\\
F_H(\gamma W \to ZW) \ &= \ \frac{c_w}{s_W}\, F_H(\gamma W \to \gamma W) && \text{(B.19)}\\
F_H(ZW \to \gamma W) \ &= \ F_H(\gamma W \to ZW) && \text{(B.20)}
\end{aligned}$$

$$\begin{aligned}
F_H(ZW \to ZW) = {} & \frac{g_2^2 M_W^2}{c_W^2}\,(\epsilon_1\epsilon_3)(\epsilon_2\epsilon_4)\ \left[\frac{1}{t - M_W^2} - \frac{1}{D_H(t)}\right] + \\
& + 2g_2^2 d \left[\ \frac{1}{c_W^2}(\epsilon_1\epsilon_3)V_{24} + c_W^2(\epsilon_2\epsilon_4)V_{13} - \frac{2d}{M_W^2} c_W^2 V_{13}V_{24}\ \right] \frac{1}{D_H(t)}
\end{aligned} \tag{B.21}$$

$$F_H(\gamma\gamma \to WW) = \frac{2g_2^2 s_W^2 d}{M_W} V_{12} \left[\ (\epsilon_3\epsilon_4) M_W - \frac{2d}{M_W} V_{34}\ \right] \frac{1}{D_H(s)} \tag{B.22}$$

$$F_H(\gamma Z \to WW) = \frac{c_W}{s_W} F_H(\gamma\gamma \to WW) \tag{B.23}$$

$$\begin{aligned}
F_H(ZZ \to WW) = {} & \frac{g_2^2 M_W^2}{c_W^2}\,(\epsilon_1\epsilon_2)(\epsilon_3\epsilon_4)\ \left[\frac{1}{s - M_W^2} - \frac{1}{D_H(s)}\right] + \\
& + 2g_2^2 d \left[\ \frac{1}{c_W^2}(\epsilon_1\epsilon_2)V_{34} + c_W^2(\epsilon_3\epsilon_4)V_{12} - \frac{2d}{M_W^2} c_W^2 V_{12}V_{34}\ \right] \frac{1}{D_H(s)}
\end{aligned} \tag{B.24}$$

$$\begin{aligned}
F_H(ZZ \to ZZ) = {} & \frac{g_2^2 M_W^2}{c_W^4} \Big\{\ (\epsilon_1\epsilon_2)(\epsilon_3\epsilon_4)\ \left[\frac{1}{s - M_W^2} - \frac{1}{D_H(s)}\right] + \\
& + (\epsilon_1\epsilon_3)(\epsilon_2\epsilon_4)\ \left[\frac{1}{t - M_W^2} - \frac{1}{D_H(t)}\right] + (\epsilon_1\epsilon_4)(\epsilon_2\epsilon_3)\ \left[\frac{1}{u - M_W^2} - \frac{1}{D_H(u)}\right]\Big\} + \\
& + 2dg_2^2 \Big\{\Big[\ (\epsilon_3\epsilon_4)V_{12} + (\epsilon_1\epsilon_2)V_{34} - \frac{2dc_W^4}{M_W^2} V_{12}V_{34}\ \Big] \frac{1}{D_H(s)} + \\
& + \left[\ (\epsilon_2\epsilon_4)V_{13} + (\epsilon_1\epsilon_3)V_{24} - \frac{2dc_W^4}{M_W^2} V_{13}V_{24}\ \right] \frac{1}{D_H(t)} + \\
& + \left[\ (\epsilon_1\epsilon_4)V_{23} + (\epsilon_2\epsilon_3)V_{14} - \frac{2dc_W^4}{M_W^2} V_{14}V_{23}\ \right] \frac{1}{D_H(u)}\ \Big\}
\end{aligned} \tag{B.25}$$

$$F_H(W^+W^- \to W^+W^-) = g_2^2 M_W^2 \left\{ (\epsilon_1\epsilon_2)(\epsilon_3\epsilon_4) \left[\frac{1}{s - M_W^2} - \frac{1}{D_H(s)} \right] + \right.$$
$$\left. + (\epsilon_1\epsilon_3)(\epsilon_2\epsilon_4) \left[\frac{1}{t - M_W^2} - \frac{1}{D_H(t)} \right] \right\}$$
$$+ 2dg_2^2 \left\{ \left[(\epsilon_3\epsilon_4)V_{12} + (\epsilon_1\epsilon_2)V_{34} - \frac{2d}{M_W^2} V_{12}V_{34} \right] \frac{1}{D_H(s)} + \right.$$
$$\left. + \left[(\epsilon_2\epsilon_4)V_{13} + (\epsilon_1\epsilon_3)V_{24} - \frac{2d}{M_W^2} V_{13}V_{24} \right] \frac{1}{D_H(t)} \right\} \tag{B.26}$$

$$F_H(W^+W^+ \to W^+W^+) = g_2^2 M_W^2 \left\{ (\epsilon_1\epsilon_4)(\epsilon_2\epsilon_3) \left[\frac{1}{u - M_W^2} - \frac{1}{D_H(u)} \right] + \right.$$
$$\left. + (\epsilon_1\epsilon_3)(\epsilon_2\epsilon_4) \left[\frac{1}{t - M_W^2} - \frac{1}{D_H(t)} \right] \right\}$$
$$+ 2dg_2^2 \left\{ \left[(\epsilon_1\epsilon_4)V_{23} + (\epsilon_2\epsilon_3)V_{14} - \frac{2d}{M_W^2} V_{14}V_{23} \right] \frac{1}{D_H(u)} + \right.$$
$$\left. + \left[(\epsilon_2\epsilon_4)V_{13} + (\epsilon_1\epsilon_3)V_{24} - \frac{2d}{M_W^2} V_{13}V_{24} \right] \frac{1}{D_H(t)} \right\} \tag{B.27}$$

$$F_H(W^+W^- \to \gamma\gamma) = \frac{2g_2^2 s_W^2 d}{M_W} V_{34} \left[(\epsilon_1\epsilon_2)M_W - \frac{2d}{M_W} V_{12} \right] \frac{1}{D_H(s)} \tag{B.28}$$

$$\begin{aligned} F_H(W^+W^- \to \gamma Z) &= \frac{c_W}{s_W} F_H(W^+W^- \to \gamma\gamma) = \\ &= \frac{2g_2^2 s_W c_W d}{M_W} V_{34} \left[(\epsilon_1\epsilon_2)M_W - \frac{2d}{M_W} V_{12} \right] \frac{1}{D_H(s)} \end{aligned} \tag{B.29}$$

$$F_H(W^+W^- \to ZZ) = g_2^2 \frac{M_W^2}{c_W^2} (\epsilon_1\epsilon_2)(\epsilon_3\epsilon_4) \left[\frac{1}{s - M_W^2} - \frac{1}{D_H(s)} \right] +$$
$$+ 2g_2^2 d \left[\frac{1}{c_W^2} (\epsilon_3\epsilon_4)V_{12} + c_W^2 (\epsilon_1\epsilon_2)V_{34} - \frac{2dc_W^2}{M_W^2} V_{12}V_{34} \right] \frac{1}{D_H(s)} \tag{B.30}$$

$$F_H(\gamma\gamma \to \gamma\gamma) = -\frac{4g_2^2 s_W^4 d^2}{M_W^2} \left\{ \frac{V_{12}V_{34}}{D_H(s)} + \frac{V_{13}V_{24}}{D_H(t)} + \frac{V_{14}V_{23}}{D_H(u)} \right\} \tag{B.31}$$

$$F_H(\gamma Z \to \gamma\gamma) = \frac{c_W}{s_W} F_H(\gamma\gamma \to \gamma\gamma) \tag{B.32}$$

$$F_H(ZZ \to \gamma\gamma) = \frac{2g_2^2 s_W^2 d}{M_W} \left\{ \frac{Z_{12}V_{34}}{D_H(s)} - \frac{2c_W^2 d}{M_W} \left[\frac{V_{13}V_{24}}{D_H(t)} + \frac{V_{14}V_{23}}{D_H(u)} \right] \right\} \tag{B.33}$$

$$F_H(\gamma\gamma \to \gamma Z) = \frac{c_W}{s_W} F_H(\gamma\gamma \to \gamma\gamma) \tag{B.34}$$

$$F_H(\gamma Z \to \gamma Z) = \frac{2g_2^2 s_W^2 d}{M_W} \left\{ \frac{Z_{24} V_{13}}{D_H(t)} - \frac{2c_W^2 d}{M_W} \left[\frac{V_{12} V_{34}}{D_H(s)} + \frac{V_{14} V_{23}}{D_H(u)} \right] \right\} \tag{B.35}$$

$$F_H(ZZ \to \gamma Z) = \frac{2g_2^2 s_W d}{M_W} \left\{ \frac{Z_{12} V_{34}}{D_H(s)} + \frac{V_{13} Z_{24}}{D_H(t)} + \frac{Z_{14} V_{23}}{D_H(u)} \right\} \tag{B.36}$$

$$F_H(\gamma\gamma \to ZZ) = \frac{2g_2^2 s_W^2 d}{M_W} \left\{ \frac{Z_{34} V_{12}}{D_H(s)} - \frac{2c_W^2 d}{M_W} \left[\frac{V_{13} V_{24}}{D_H(t)} + \frac{V_{14} V_{23}}{D_H(u)} \right] \right\} \tag{B.37}$$

$$F_H(\gamma Z \to ZZ) = \frac{2g_2^2 s_W c_W d}{M_W} \left\{ \frac{V_{12} Z_{34}}{D_H(s)} + \frac{V_{13} Z_{24}}{D_H(t)} + \frac{V_{14} Z_{23}}{D_H(u)} \right\} \tag{B.38}$$

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Figure captions

Fig.1 Invariant mass distribution in W^+Z production via $W^+Z \to W^+Z$. Standard Model (solid), including $\mathcal{O}_W$ ($\lambda = 0.01$ dashed, $\lambda = -0.01$ short dashed), including $\mathcal{O}_{UW}$ ($d = \pm 0.1$ long dashed).

Fig.2 Invariant mass distribution in ZZ production via various fusion processes. Standard Model(solid), including $\mathcal{O}_W$ ($\lambda = 0.01$ dashed, $\lambda = -0.01$ short dashed), including $\mathcal{O}_{UW}$ ($d = \pm 0.1$ long dashed).

Fig.3 Invariant mass distribution for various final states summing over all possible initial states. Standard Model (solid), including $\mathcal{O}_W$ ($\lambda = 0.01$ dashed). The results for $\lambda = -0.01$ are similar to those for $\lambda = 0.01$.

Fig.4 Invariant mass distribution for various final states summing all possible initial states. Standard Model (solid), including $\mathcal{O}_{UW}$ ($d = \pm 0.1$ long dashed).

Fig.5 Ratios of invariant mass distributions $WZ/\gamma\gamma$ and $WZ/W\gamma$. Standard Model (solid), including $\mathcal{O}_W$ ($\lambda = 0.01$ dashed, $\lambda = -0.01$ short dashed), including $\mathcal{O}_{UW}$ ($d = \pm 0.1$ long dashed).

Fig.6 Ratios of invariant mass distributions WZ/ZZ and $W\gamma/ZZ$. Standard Model (solid), including $\mathcal{O}_W$ ($\lambda = 0.01$ dashed, $\lambda = -0.01$ short dashed), including $\mathcal{O}_{UW}$ ($d = \pm 0.1$ long dashed).